

## 5-5 Differences of Two Squares

**Objective:** To simplify products of the form  $(a + b)(a - b)$  and to factor differences of two squares.

### Vocabulary

**Product of the Sum and Difference of Two Numbers**

$$(a + b)(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$$

**Difference of Two Squares**

$$a^2 - b^2 = (a + b)(a - b)$$

**Example 1** Write each product as a binomial.

a.  $(x + 2)(x - 2)$     b.  $(2n + 3)(2n - 3)$

**Solution** These products fit the form  $(a + b)(a - b)$ , so each binomial is of the form  $a^2 - b^2$ .

a.  $(x + 2)(x - 2) = (x)^2 - (2)^2$   
 $= x^2 - 4$

b.  $(2n + 3)(2n - 3) = (2n)^2 - (3)^2$   
 $= 4n^2 - 9$

**Write each product as a binomial.**

1.  $(a + 3)(a - 3)$   $a^2 - 9$

2.  $(4 - x)(4 + x)$   $16 - x^2$

3.  $(x + 5)(x - 5)$   $x^2 - 25$

4.  $(9 - x)(9 + x)$   $81 - x^2$

5.  $(5a + 2)(5a - 2)$   $25a^2 - 4$

6.  $(7a - 2)(7a + 2)$   $49a^2 - 4$

7.  $(4 + 3x)(4 - 3x)$   $16 - 9x^2$

8.  $(6 - 5x)(6 + 5x)$   $36 - 25x^2$

9.  $(3 - 5x)(3 + 5x)$   $9 - 25x^2$

10.  $(8x + 7)(8x - 7)$   $64x^2 - 49$

**Example 2** Write each product as a binomial.

a.  $(a^2 - 3b)(a^2 + 3b)$     b.  $(xy + z)(xy - z)$

**Solution** These products fit the form  $(a + b)(a - b)$ , so each binomial is of the form  $a^2 - b^2$ .

a.  $(a^2 - 3b)(a^2 + 3b) = (a^2)^2 - (3b)^2$   
 $= a^4 - 9b^2$

b.  $(xy + z)(xy - z) = (xy)^2 - z^2$   
 $= x^2y^2 - z^2$

**Write each product as a binomial.**

11.  $9x^2 - 16y^2$

12.  $4u^2 - v^2$

13.  $x^4 - 64y^2$

11.  $(3x + 4y)(3x - 4y)$

12.  $(2u + v)(2u - v)$

13.  $(x^2 - 8y)(x^2 + 8y)$

14.  $(x^2 - 3y^2)(x^2 + 3y^2)$   
 $x^4 - 9y^4$

15.  $(2a^2 + 5b^2)(2a^2 - 5b^2)$   
 $4a^4 - 25b^4$

16.  $(ab - 2c)(ab + 2c)$   
 $a^2b^2 - 4c^2$

## 5-5 Differences of Two Squares (continued)

**Example 3** Multiply. Use the pattern  $(a + b)(a - b) = a^2 - b^2$ .

a.  $58 \cdot 62$

b.  $93 \cdot 87$

**Solution** a.  $58 \cdot 62 = (60 - 2)(60 + 2)$     b.  $93 \cdot 87 = (90 + 3)(90 - 3)$   
 $= 3600 - 4$      $= 8100 - 9$   
 $= 3596$      $= 8091$

**Multiply.** Use the pattern  $(a + b)(a - b) = a^2 - b^2$ .

17.  $16 \cdot 24$  **384**

18.  $27 \cdot 33$  **891**

19.  $53 \cdot 47$  **2491**

20.  $35 \cdot 45$  **1575**

21.  $41 \cdot 39$  **1599**

22.  $92 \cdot 88$  **8096**

23.  $104 \cdot 96$  **9984**

24.  $60 \cdot 140$  **8400**

**Example 4** Factor: a.  $a^2 - 16$     b.  $9 - 4b^2$     c.  $25a^2 - 36x^6$

**Solution** Use the pattern  $a^2 - b^2 = (a + b)(a - b)$

a.  $a^2 - 16 = a^2 - 4^2$   
 $= (a + 4)(a - 4)$

b.  $9 - 4b^2 = 3^2 - (2b)^2$   
 $= (3 + 2b)(3 - 2b)$

c.  $25a^2 - 36x^6 = (5a)^2 - (6x^3)^2$   
 $= (5a + 6x^3)(5a - 6x^3)$

**Factor.** You may use a calculator or a table of squares.

25.  $b^2 - 16$   $(b + 4)(b - 4)$     26.  $f^2 - 81$   $(f + 9)(f - 9)$     27.  $36 - x^2$   $(6 + x)(6 - x)$

28.  $9e^2 - 16$   $(3e + 4)(3e - 4)$     29.  $49n^2 - 1$

30.  $4a^2 - 9$   $(2a + 3)(2a - 3)$

31.  $a^4 - 36$   $(a^2 + 6)(a^2 - 6)$     32.  $49b^2 - 16c^2$

33.  $100 - 121r^2$

34.  $121 - y^2$   $(11 + y)(11 - y)$     35.  $25u^2 - 36$

36.  $16x^2 - 225$

29.  $(7n + 1)(7n - 1)$

33.  $(10 + 11r)(10 - 11r)$

32.  $(7b + 4c)(7b - 4c)$

36.  $(4x + 15)(4x - 15)$

35.  $(5u + 6)(5u - 6)$

### Mixed Review Exercises

**Simplify.**

$8z^2 + 2z$

$x^2 - x - 20$

$-4m^2 + 9m - 6$

1.  $5z(z - 2) + 3z(z + 4)$

2.  $(x + 4)(x - 5)$

3.  $-3(m + 2) - 4m(m - 3)$

4.  $\frac{36a^5b^2}{9a^3} \cdot 4a^2b^2$

5.  $\frac{15a + 5}{5} \cdot 3a + 1$

6.  $\frac{18n^2x}{6nx} \cdot 3n$

7.  $(a + 2)(2a - 1)$

8.  $(3b + 2)(b - 1)$

9.  $(4x)^2 \left(\frac{1}{4}\right)^2 x \cdot x^3$

10.  $\frac{12y^3 + 28y^2 - 8y}{4y}$

11.  $\frac{30x^3 + 45x^2 - 15x}{15x}$

12.  $\frac{24x^3y^4z}{3x^3y^3z} \cdot 8y$

$3y^2 + 7y - 2$

$2x^2 + 3x - 1$